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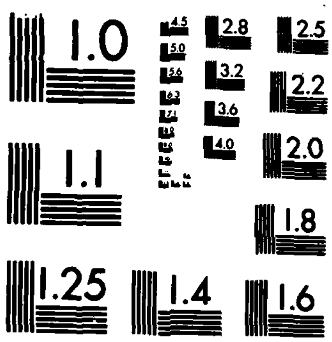

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SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS

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ABSTRACT

~~We~~ This document give a new proof of Véron's result concerning the classification of isolated singularities for the equation $-\Delta u + u^p = 0$ (delta, sub p). ~~We~~ The authors also establish that the singular behavior at a point can be prescribed and determines uniquely the solution (under fixed boundary conditions). Keywords: nonlinear elliptic equations; Scaling; applied mathematics; mathematical analysis.

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SIGNIFICANCE AND EXPLANATION

Nonlinear elliptic equations with isolated singularities occur in physical problems with point sources. A good example is the Thomas-Fermi theory of atoms and molecules which leads to the equation $-\Delta u + u^{3/2} = 0$ in $\mathbb{R}^3 \setminus \bigcup_{i=1}^{k} \{a_i\}$.

The points $\{a_i\}$ correspond to the location of positive nuclei of charge m_i. Near a_i the solution u has a singular behavior equivalent to $m_i E(x - a_i)$ where E is the fundamental solution of $-\Delta$, i.e. $E(x) = (4\pi|x|)^{-1}$. A striking result of L. Véron provides a complete classification of all singular solutions, and shows that isolated singularities of nonlinear problems are quite rigid. In this paper we present a new proof of Véron's result based on a simple scaling argument. We also establish that the singular behavior at a point can be prescribed very much like a boundary condition and determines uniquely the solution.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS

Haïm Brezis and Luc Oswald

Dedicated to Jim Serrin on his sixtieth birthday

1. Introduction

Let $B_R = \{x \in R^N;\ |x| < R\}$ with $N > 2$. Consider a function u which satisfies

$$(1)\qquad \begin{cases} u \in C^2(B_R\setminus\{0\}), \quad u > 0 \text{ on } B_R\setminus\{0\}, \\ -\Delta u + u^p = 0 \text{ on } B_R\setminus\{0\}. \end{cases}$$

We are concerned with the behavior of u near $x = 0$. There are two distinct cases:

1) When $p > N/(N-2)$ and $(N > 3)$ it has been shown by Brezis - Véron [9] that u must be smooth at 0 (See also Baras-Pierre [1] for a different proof). In other words, isolated singularities are removable.

2) When $1 < p < N/(N-2)$ there are solutions of (1) with a singularity at $x = 0$. Moreover all singular solutions have been classified by Véron [22]. We recall his result:

Theorem 1 Assume $1 < p < N/(N-2)$ and u satisfies (1). Then one of the following holds:

(i) either u is smooth at 0,

(ii) or $\lim_{x\to 0} u(x)/E(x) = c$ where c is a constant which can take any value in the interval $(0,\infty)$,

(iii) or $\lim_{x\to 0} \left|u(x) - \ell(p,N)|x|^{-2/(p-1)}\right| = 0$.

Here $E(x)$ denotes the fundamental solution of $-\Delta$ and $\ell = \ell(p,N)$ is the (unique) positive constant C such that $C|x|^{-2/(p-1)}$ satisfies (1) - more precisely

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$$\ell = \ell(p,N) = \left[\frac{2}{(p-1)}\left(\frac{2p}{p-1} - N\right)\right]^{1/(p-1)} .$$

We shall first present a proof of Theorem 1 which is simpler than the original proof of Vêron. In particular, it does not make use of Fowler's results [10] for the Emden differential equation. Instead, it relies on some simple scaling argument (see the proof of Lemma 5) which is similar to the one used by Kamin-Peletier [12] for parabolic equations.

Next, we emphasize that a singular behavior such as (ii) or (iii) can be prescribed together with a boundary condition, and these determine uniquely the solution.

More precisely, let Ω be a smooth bounded domain in $\mathbb{R}^N$ with $0 \in \Omega$ and let $\varphi > 0$ be a smooth function defined on $\partial\Omega$. We consider the problem

$$(2) \qquad \begin{cases} u \in C^2(\bar{\Omega}\setminus\{0\}), \quad u > 0 & \text{on } \Omega \setminus \{0\} , \\ -\Delta u + u^p = 0 & \text{on } \Omega \\ \qquad u = \varphi & \text{on } \partial\Omega . \end{cases}$$

Theorem 2 Assume $1 < p < N/(N-2)$. Then:

(i) There is a unique solution u_0 of (2) which belongs to $C^2(\bar{\Omega})$.

(ii) Given any constant $c \in (0, +\infty)$ there is a unique solution u_c of (2) which satisfies

$$\lim_{x \to 0} u(x)/E(x) = c .$$

(iii) There is a unique solution u_∞ of (2) which satisfies

$$\lim_{x \to 0} |x|^{2/(p-1)} u(x) = \ell(p,N)$$

In addition, $\lim\limits_{c \downarrow 0} u_c = u_0$ and $\lim\limits_{c \uparrow \infty} = u_\infty$.

Singular solutions of (1) occur in the Thomas-Fermi theory with $N = 3$ and $p = 3/2$ (see e.g. [13] for a detailed exposition). Other results dealing with singular solutions of nonlinear elliptic equations have been obtained by a number of authors: J. Serrin [20], [21], Véron and Vazquez (See the exposition in [23]), P. L. Lions [14], W. M. Ni-J. Serrin [16]. Semilinear parabolic equations with isolated singularities have been considered by Brezis - Friedman [5], Brezis - Peletier - Terman [8], Kamin - Peletier [12], Oswald [18].

2. Some preliminary facts

We recall some known results dealing with functions u satisfying (1).

Set $\alpha = 2/(p-1)$ (for $1 < p < \infty$).

Lemma 1 Assume $u \in C^2(B_R)$ satisfies (1).

Then

$$u(0) \le C(p,N)/R^\alpha$$

where $C(p,N)$ is defined by $C(p,N) = \mathrm{Max}\,\{2\alpha N,\ 4\alpha(\alpha+1)\}^{1/(p-1)}$.

The proof of Lemma 1 uses a comparision function U of the same type as in Osserman [17] (or Loewner - Nirenberg [15]), namely set

$$U(x) = \frac{C(p,n)\ R^\alpha}{(R^2 - |x|^2)^\alpha} \quad \text{on } B_R .$$

A direct computation shows that

$$-\Delta U + U^p \ge 0 \qquad \text{on } B_R .$$

By the maximum principle we see that

$$u \le U \quad \text{on } B_R$$

and in particular $u(0) \le U(0)$.

Lemma 2 Assume u satisfies (1) with $1 < p < N/(N-2)$. Then, for $0 < |x| < R/2$, we have

$$u(x) \le \frac{\ell(p,N)}{|x|^\alpha}\left(1 + \frac{C(p,N)}{\ell(p,N)}\left(\frac{|x|}{R}\right)^\beta\right)$$

where $\beta = 2\alpha + 2 - N > \alpha$.

Lemma 2 is established in Brezis - Lieb [6] (proposition A.4) for the special case where $N = 3$ and $p = 3/2$. The proof in the general case is just the same.

Lemma 3 Assume $1 < p < N/(N-2)$ and let $c > 0$ be a constant. Then, there is a unique function u satisfying

$$
(3)\qquad \begin{cases} u \in L^p(\mathbb{R}^N) \cap C^2(\mathbb{R}^N\setminus\{0\}) , \\ u > 0 \quad \text{on} \quad \mathbb{R}^N\setminus\{0\} , \\ -\Delta u + u^p = c\delta \quad \text{on} \quad \mathbb{R}^N \end{cases}
$$

We set $u = W_c$.

Lemma 3, as well as Lemma 4 below, are due to Benilan - Brezis (unpublished); the ingredients for the proofs may be found in [2], [3], [4] (and also [1] and [11]).

Finally, we assume that Ω is a smooth bounded domain in $\mathbb{R}^N$ with $0 \in \Omega$ and that $\varphi > 0$ is a smooth function defined as $\partial\Omega$.

<u>Lemma 4</u> Assume $1 < p < N/(N-2)$ and let $c > 0$ be a constant. Then, there is a unique function u satisfying

$$
(4)\qquad \begin{cases} u \in L^p(\Omega) \cap C^2(\bar{\Omega}\setminus\{0\}) \\ u > 0 \quad \text{on} \quad \Omega\setminus\{0\} \\ -\Delta u + u^p = c\delta \quad \text{on} \quad \Omega \\ \quad u = \varphi \quad \text{on} \quad \partial\Omega . \end{cases}
$$

3. A Scaling Argument

An important step in the proof of Theorem 1 is the following

Lemma 5 Assume $1 < p < N/(N-2)$. Then we have

$$\lim_{c\uparrow\infty} W_c(x) = \ell|x|^{-\alpha} \equiv W_\infty(x) .$$

Proof It is clear (by comparison) that $W_c(x)$ is a nondecreasing function of c. Moreover we have

$$W_c(x) \leq \ell|x|^{-\alpha}$$

(by letting $R \to \infty$ in Lemma 2). Therefore $\lim_{c\uparrow\infty} W_c(x) = W_\infty(x)$ exists pointwise (for $x \neq 0$) and $W_\infty(x) \leq \ell|x|^{-\alpha}$. The uniqueness of the solution of (3) implies that $W_c(x)$ is radial and so is $W_\infty(x)$. Next, we observe that the function

$$u(x) = k^\alpha W_c(kx) \qquad (k > 0)$$

satisfies

$$-\Delta u(x) + u^p(x) = k^{\alpha p} c\delta(kx) = k^{\alpha p-N} c\delta(x) .$$

It follows, again by uniqueness, that

$$k^\alpha W_c(kx) = W_{ck^{\alpha p-N}}(x) .$$

As $c\uparrow\infty$ we see that

$$k^\alpha W_\infty(kx) = W_\infty(x) .$$

Choosing $k = 1/|x|$ we obtain

$$W_\infty(x) = W_\infty\left(\frac{x}{|x|}\right)|x|^{-\alpha} = C|x|^{-\alpha}$$

where $C > 0$ is some constant.

Finally we note that since

$$-\Delta W_c + W_c^p = 0 \quad \text{in} \quad \mathcal{D}'(\mathbb{R}^N\setminus\{0\})$$

and

$$W_c \to W_\infty \quad \text{in} \quad L^p_{\ell oc}(\mathbb{R}^N\setminus\{0\}),$$

it follows that

$$-\Delta W_\infty + W_\infty^p = 0 \quad \text{in} \quad \mathcal{D}'(\mathbb{R}^N \setminus \{0\}).$$

This determines the value of the constant C to be $C = \ell$.

There is a similar result in balls: Set $u = V_c$ to be the unique solution of problem (4) with $\Omega = B_R$.

<u>Lemma 6</u> Assume $1 < p < N/(N-2)$. Then we have $V_\infty(x) \equiv \lim_{c\uparrow\infty} V_c(x)$ exists pointwise on $B_R \setminus \{0\}$ and moreover

$$W_\infty(x) - \ell R^{-\alpha} \leq V_\infty(x) \leq W_\infty(x) \quad \text{on} \quad B_R .$$

<u>Proof</u> It is again clear (by comparison) that $V_c(x)$ is a nondecreasing function of c. Also we have

(5) $$0 \leq V_c(x) \leq W_c(x) .$$

It follows from (4) and (5) that

$$-\Delta(W_c - V_c) \leq 0 \quad \text{on} \quad B_R ,$$

and consequently $\sup\limits_{B_R} (W_c - V_c) \leq \sup\limits_{\partial B_R} (W_c - V_c) \leq \sup\limits_{\partial B_R} W_\infty = \ell R^{-\alpha}$.

The conclusion follows by letting $c \to \infty$.

4. Proof of Theorem 1

Throughout this section we suppose $1 < p < N/(N-2)$. Assume u satisfies (1) and set

$$c = \limsup_{x \to 0} u(x)/E(x) .$$

We distinguish three cases:

Case (i) $c = 0$

Case (ii) $0 < c < \infty$

Case (iii) $c = \infty$.

Cases (i) and (ii).

Here, the main ingredient is the following:

Lemma 7 In cases (i) and (ii) the function u belongs to $L^p_{\ell oc}(B_R)$ and satisfies

$$-\Delta u + u^p = c_0 \delta \quad \text{in} \quad \mathcal{D}'(B_R)$$

for some constant c_0.

Proof It is clear that $u \in L^p_{\ell oc}(B_R)$ since $E \in L^p_{\ell oc}(B_R)$ and $c < \infty$.

We now use the same argument as in [7]: set

$$T = -\Delta u + u^p \in \mathcal{D}'(B_R) .$$

Since the support of T is contained in $\{0\}$, it follows from a classical result about distributions (see [19]) that

$$T = \sum_{0 \le |\alpha| \le m} c_\alpha D^\alpha(\delta) . \tag{6}$$

We claim that $c_\alpha = 0$ when $|\alpha| \geq 1$. Indeed let $\zeta \in \mathcal{D}(B_R)$ be any fixed function such that $(-1)^{|\alpha|} D^\alpha \zeta(0) = c_\alpha$ for every α with $|\alpha| \le m$. Multiplying (6) through by $\zeta_\varepsilon(x) = \zeta(x/\varepsilon)$ we obtain

$$- \int u \Delta \zeta_\varepsilon + \int u^p \zeta_\varepsilon = \sum_{0 \le |\alpha| \le m} c_\alpha^2 \, \varepsilon^{-|\alpha|} .$$

An easy computation - using the estimate $u \le CE$ - shows that

$$\begin{cases} \left|\int u\, \Delta\zeta_\varepsilon\right| \le C & \text{when } N \ge 3 \\ \left|\int u\, \Delta\zeta_\varepsilon\right| \le C|\log \varepsilon| + C & \text{when } N = 2\ . \end{cases}$$

Since $\int u^p \zeta_\varepsilon \to 0$ as $\varepsilon \to 0$, we conclude that $c_\alpha = 0$ for $|\alpha| \ge 1$. Therefore we obtain

$$-\Delta u + u^p = c_0\delta \quad \text{in} \quad \mathcal{D}'(B_R)$$

We conclude the proof of Theorem 1 in cases (i) and (ii) with the help of the following:

<u>Lemma 8</u> Assume $u \in C^2(B_R\setminus\{0\}) \cap L^p_{\ell oc}(B_R)$ satisfies

$$\begin{cases} u \ge 0 & \text{on } B_R\ , \\ -\Delta u + u^p = c_0\delta & \text{in } \mathcal{D}'(B_R) \end{cases}$$

for some constant c_0.

We have

(i) if $c_0 = 0$, then u is smooth on B_R ,

(ii) if $c_0 \ne 0$, then $\lim\limits_{x\to 0} u(x)/E(x) = c_0$.

<u>Proof</u>

(i) Assume $c_0 = 0$. Since u is subharmonic it follows that $u \in L^\infty_{\ell oc}(B_R)$ and thus $\Delta u \in L^\infty_{\ell oc}(B_R)$. We deduce that $u \in C^1(B_R)$ and then $u \in C^2(B_R)$. In fact $u \in C^\infty(B_R)$ since, by the strong maximum principle, we have either $u \equiv 0$ or $u > 0$ or B_R.

(ii) Assume $c_0 \ne 0$. By the maximum principle we have

$$u \le c_0E + C \quad \text{on} \quad B_{R/2}$$

and therefore

$$\begin{aligned} -\Delta u &\ge c_0\delta - (c_0E + C)^p \\ &\ge c_0\delta - C(E^p + 1) \quad \text{on} \quad B_{R/2} \end{aligned}$$

An elementary computation leads to

$$u(x) \geq c_0 E - o(E) \quad \text{as} \quad x \to 0 \,.$$

and we conclude that $\lim\limits_{x \to 0} u(x)/E(x) = c_0$.

Remark 1 Assume $c_0 \neq 0$. The argument above provides in fact an estimate for $|u - c_0 E|$ as $x \to 0$. More precisely we have

a) If $N = 2$ and $1 < p < \infty$ or $N = 3$ and $1 < p < 2$, then

$$|u - c_0 E| \leq C \quad \text{on} \quad B_{R/2}$$

b) If $N = 3$ and $p = 2$, then

$$|u(x) - c_0 E(x)| \leq C(|\log|x|| + 1) \quad \text{on} \quad B_{R/2}$$

c) If $N = 3$ and $2 < p < 3$ or $N \geq 4$ and $1 < p < N/(N-2)$ then

$$|u(x) - c_0 E(x)| \leq C|x|^{2-(N-2)p} \quad \text{on} \quad B_{R/2}$$

and consequently

$$\left|\frac{u(x)}{E(x)} - c_0\right| \leq C\,|x|^{\nu} \quad \text{on} \quad B_{R/2}$$

with $\nu = N - (N-2)p > 0$.

Proof of Theorem 1 in the case (iii)

We first recall a result of Véron [22] (Lemma 1.5):

Lemma 9 Assume u satisfies (1). Then, there is a constant C (depending only as p and N) such that

$$\sup_{|x|=r} u(x) \leq C \inf_{|x|=r} u(x) \quad \text{for} \quad 0 < r < R/2.$$

The conclusion of Lemma 9 is a simple consequence of Harnack's inequality and the estimate of Lemma 1 - see [22] for the details.

We may now complete the proof of Theorem 1 with the help of the following:

Lemma 10 Assume u satisfies (1) and $\limsup\limits_{x \to 0} u(x)/E(x) = \infty$. Then

$$|u(x) - \ell|x|^{-\alpha}| \leq C|x|^{\gamma} \quad \text{on} \quad B_{R/2}$$

for some constants $C = C(p,N,R)$ and $\gamma = \gamma(p,N) > 0$.

Proof By Lemma 2 we already have the estimate

$$u(x) \le \ell|x|^{-\alpha} + C|x|^{\gamma} \quad \text{on} \quad B_{R/2}$$

with

$$\gamma = \beta - \alpha = \alpha + 2 - N > 0.$$

We now establish an estimate from below. Let $x_n \to 0$ be such that $\lim u(x_n)/E(x_n) = \infty$. Set $r_n = |x_n|$, so that we obtain from Lemma 9

(7) $$\inf_{|x|=r_n} u(x)/E(x) \underset{n\to\infty}{\to} \infty .$$

We recall that V_c is the unique solution of (4) when $\Omega = B_R$, so that

$V_c \le cE$ on B_R.

Given any constant $c > 0$, we see (by (7)) that

$$u(x) \ge cE(x) \quad \text{for} \quad |x| = r_n \quad \text{and} \quad n \quad \text{large enough} .$$

Therefore we obtain

$$u(x) \ge V_c(x) \quad \text{for} \quad |x| = r_n \quad \text{and} \quad n \quad \text{large enough} .$$

Applying the maximum principle in the domain $\{x \in R^N;\ r_n < |x| < R\}$ we find that

$$u(x) \ge V_c(x) \quad \text{for} \quad r_n < |x| < R \quad \text{and} \quad n \quad \text{large enough} .$$

As $n \to \infty$ we conclude that

$$u(x) \ge V_c(x) \quad \text{on} \quad B_R\backslash\{0\}$$

and as $c \to \infty$ we see that

$$u(x) \ge V_\infty(x) \quad \text{on} \quad B_R\backslash\{0\} .$$

In Lemma 6 we had the estimate

$$V_\infty(x) \ge \ell(|x|^{-\alpha} - R^{-\alpha}) .$$

However it is not good enough to deduce conclusion (iii) of Theorem 1. We need a better estimate from below for $V_\infty(x)$; we claim that

(8) $$V_\infty(x) \ge \ell|x|^{-\alpha} \left(1 - (\frac{|x|}{R})^{\beta}\right) \quad \text{on} \quad B_R ,$$

where β is defined in Lemma 2.

Clearly, it suffices to establish (8) for $R = 1$. The function V_∞ is radial and so we write $V_\infty(r)$. We define the function v or $(0,1)$ by the relation

$$v(r^\beta) = \ell^{-1} r^\alpha V_\infty(r)$$

so that $0 < v < 1$ on $(0,1)$, $v(1) = 0$ and $v(0) = 1$. Using the relation $-\Delta V_\infty + V_\infty^p = 0$ it is easy to deduce (as in the proof of Proposition A.4 [6]) that

$$-\beta^2 t^2 v''(t) + \ell^{p-1} v(t)(v^{p-1}(t) - 1) = 0 \quad \text{for} \quad t \in (0,1).$$

Consequently v is concave and thus we have

$$v(t) > 1 - t \qquad \forall t \in (0,1) ,$$

that is (8).

Remark 2 Véron [22] obtains in case (iii) an estimate of the form $|u(x) - \ell|x|^{-\alpha}| < C|x|^\delta$ with an exponent δ which is better than $\gamma = \beta - \alpha$.

5. Proof of Theorem 2.

Case (i) is classical.

Case (ii) The existence of a solution follows from Lemma 4 and 8.

Suppose now u satisfies (2) and $\lim_{x\to 0} u(x)/E(x) = c$. We deduce from Lemma 7 and 8 that $-\Delta u + u^p = c\delta$; uniqueness follows from Lemma 4.

Case (iii) We denote by u_c the unique solution of (4) given by Lemma 4. We claim that $u_\infty = \lim_{c\uparrow\infty} u_c$ has all the required properties.

Indeed $u_c(x)$ is a nondecreasing function of c. Fix $R > 0$ such that $2R < \mathrm{dist}(0,\partial\Omega)$. By Lemma 1 we have

$$u_c(x) \leq C(p,N)R^{-\alpha} \quad \text{for} \quad |x| = R.$$

The maximum principle applied in the region

$$\Omega_R = \{x \in \Omega; \quad |x| > R\}$$

shows that, in Ω_R,

$$u_c(x) \leq \mathrm{Max}\ \{\sup_{\partial\Omega} \varphi, \quad C(p,N)R^{-\alpha}\} .$$

Therefore $u_\infty(x) = \lim_{c\uparrow\infty} u_c(x)$ exists and u_∞ satisfies (2). By comparison on B_R we have

$$V_c \leq u_c \quad \text{on} \quad B_R$$

and as $c \to \infty$ we obtain $V_\infty \leq u_\infty$ on B_R.

It follows that $\lim_{x\to 0}|u_\infty(x) - \ell|x|^{-\alpha}| = 0$ (by Lemma 6 and Theorem 1).

We turn now the question of uniqueness. Suppose u_1 and u_2 satisfy (2) and $\lim_{x\to 0}|x|^\alpha u_i(x) = \ell$ for $i = 1, 2$. Lemma 10 implies that

$$|u_1(x) - u_2(x)| \leq C|x|^\gamma \quad \text{on} \quad B_R$$

On the other hand we have

$$-\Delta(u_1 - u_2) + u_1^p - u_2^p = 0 \quad \text{on} \quad \Omega\backslash\{0\}$$

Applying the maximum principle in Ω_R we

$$\max_{\Omega_R} |u_1 - u_2| \leq \max_{\partial B_R} |u_1 - u_2| \leq CR^\gamma$$

and then we let $R \to 0$ to conclude that $u_1 = u_2$.

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We give a new proof of Véron's result concerning the classification of isolated singularities for the equation $-\Delta u + u^p = 0$. We also establish that the singular behavior at a point can be prescribed and determines uniquely the solution (under fixed boundary conditions).

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